Appendix C: Graphing

One of the most powerful tools used for data presentation and analysis is the graph. Used properly, graphs are an important guide to understanding the results of an experiment. They are an easy way to make sense out of your data. Therefore, it is important to graph your data as you take it.

LabVIEW™ will often provide you with a graph, but on occasion you will have to make your own. Good graphing practice, as outlined below, is a way to save time and effort while solving a problem in the laboratory.

How do I make a graph?

1. Accurate graphs are drawn on graph paper. Even if you are just making a quick sketch for yourself, it will save you time and effort to use graph paper. That is why every page of your lab journal is a piece of graph paper. Make sure to graph your data as you take it. Never put off drawing a graph until the end.

2. Every graph should have a title to indicate the data it represents. In a large collection of graphs, it is difficult to keep one graph distinct from another without clear, concise titles.

3. The axes of the graph should take up at least half a page. Give yourself plenty of space so that you can see the pattern of the data as it is developing. Both axes should be labeled to show the values being graphed and their appropriate units.

4. The scales on the graphs should be chosen so that the data occupies most of the space of your graph. You do not need to include zero on your scales unless it is important to the interpretation of the graph.

5. If you have more than one set of data on a set of axes, be sure to label each set to avoid confusion later.

How do I plot data and uncertainties?

Another technique that makes data analysis easier is to record all your data in a table.

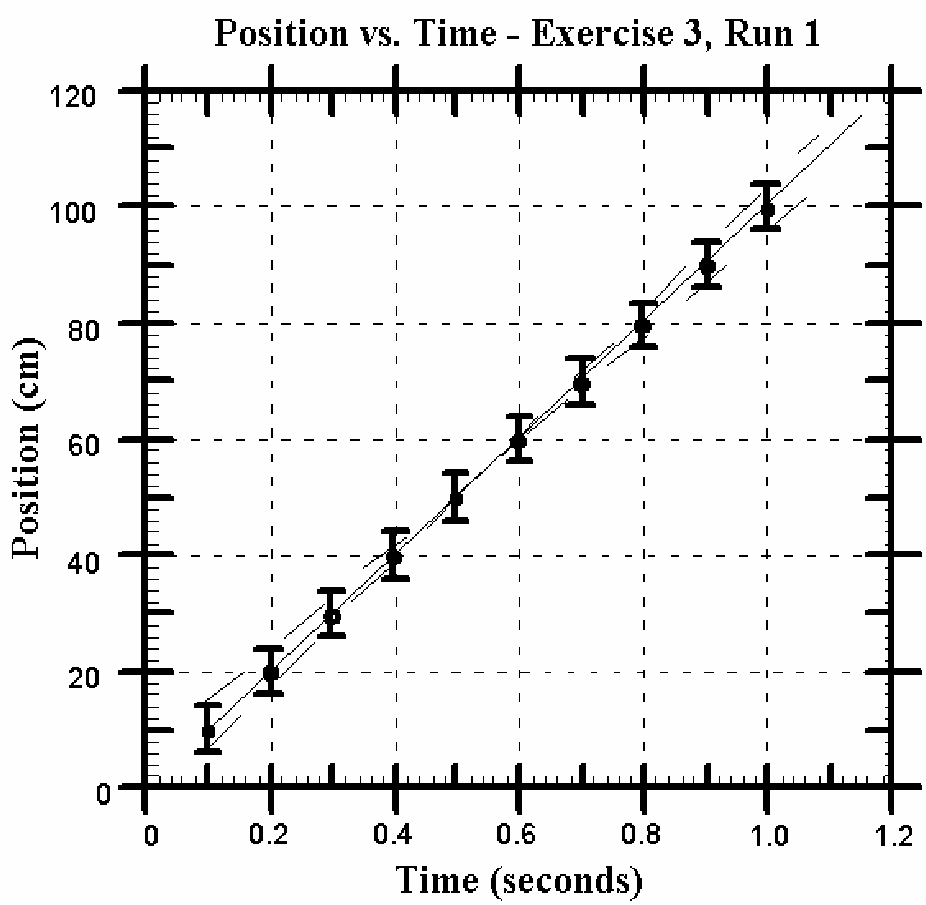
A useful data table will always include a title and column headings, so that you will not forget what all the numbers mean. The column headings should include the units of the quantities listed, and usually serve as the labels for the axes of your graph. For example, look at Table C-1 and Graph C-1 on the next page. This is a position-versus-time graph drawn for a hypothetical situation.

The uncertainty for each data point is shown on the graph as a line representing a range of possible values with the principal value at the center. The lines are called **error bars**, and they are useful in determining if your data agrees with your prediction. Any curve (function) that represents your data should pass through your error bars.

Table C-1: Position vs. Time: Exercise 3, Run 1

Time/sec	Position/cm
0.1	10 ± 5
0.2	20 ± 5
0.3	30 ± 5
0.4	39 ± 5
0.5	49 ± 5
0.6	59 ± 5
0.7	69 ± 5
0.8	79 ± 5
0.9	89 ± 5
1.0	98 ± 5

Graph C-1:



What is a best fit straight line?

If your data points appear to lie on a straight line, then use a clear straight edge and draw a line through all the error bars, making sure the line has as many principal values beneath it as above it. The line does not need to touch any of the principle values. **Do not connect the dots.** When done correctly, this straight line represents the function that best fits your data. You can read the slope and intercept from your graph. These quantities usually have important physical interpretations. Some computer programs, such as Excel or Mathematica, will determine the best straight line for your data and compute its slope, intercept, and their uncertainties. As an example, look at Graph C-1. The dashed lines are possible linear fits, but the solid line is the best fit. Once you have found the best-fit line, you should determine the slope from the graph and record its value.

How do I find the slope of a line?

The slope of a line is defined as the ratio of the change in a line's ordinate (vertical axis) to the change in the abscissa (horizontal axis), or the "rise" divided by "run." Your text explains slope. For Graph C-1, the slope of the best-fit line will be the change in the position of the object divided by the time interval for that change in position.

To find the value of the slope, look carefully at your best-fit line to find points along the line that have coordinates that you can identify. It is usually not a good idea to use your data points for these values, since your line might not pass through them exactly. For example, the best-fit line on Graph C-1 passes through the points

(0.15 sec, 15 cm) and (1.10 sec, 110 cm). This means the slope of the line is:

$$\text{Slope} = \frac{110\,\text{cm} - 15\,\text{cm}}{1.10\,\text{sec} - 0.15\,\text{sec}}$$

$$= \frac{95\ \text{cm}}{0.95\ \text{sec}}$$

$$= 100\,\frac{\text{cm}}{\text{sec}}$$

Note that the slope of a position-versus-time graph has the units of velocity.

How do I find the uncertainty in the slope of a line?

Look at the dashed lines in Graph C-1. These lines are the largest and smallest values of the slopes that can realistically fit the data. The lines run through the extremes in the uncertainties and they represent the largest and smallest *possible* slopes for lines that fit the data. You can extend these lines and compute their slopes. These are your uncertainties in the determination of the slope. In this case, it would be the uncertainty in the velocity.

How do I get the slope of a curve that is not a straight line?

The tangent to a point on a smooth curve is just the slope of the curve at that point. If the curve is not a straight line, the slope will change from point to point along the curve.

To draw a tangent line at any point on a smooth curve, draw a straight line that only touches the curve at the point of interest, without going inside the curve. Try to get an equal amount of space between the curve and the tangent line on both sides of the point of interest.

The tangent line that you draw needs to be long enough to allow you to easily determine its slope. You will also need to determine the uncertainty in the slope of the tangent line by considering all other possible tangent lines and selecting the ones with the largest and smallest slopes. The slopes of these lines will give the uncertainty in the slope of the tangent line. Notice that this is exactly like finding the uncertainty of the slope of a straight line.

How do I "linearize" my data?

A straight-line graph is the easiest graph to interpret. By seeing if the slope is positive, negative, or zero you can quickly determine the relationship between two measurements. But not all the relationships in nature produce straight-line graphs. However, if we have a theory that predicts how one measured quantity (e.g., position) depends on another (e.g., time) for the experiment, we can make the graph be a straight line. To do this, you make a graph with the appropriate function of one quantity on one axis (e.g., time squared) and the other quantity (e.g., position) on the other axis. This is called "linearizing" the data.

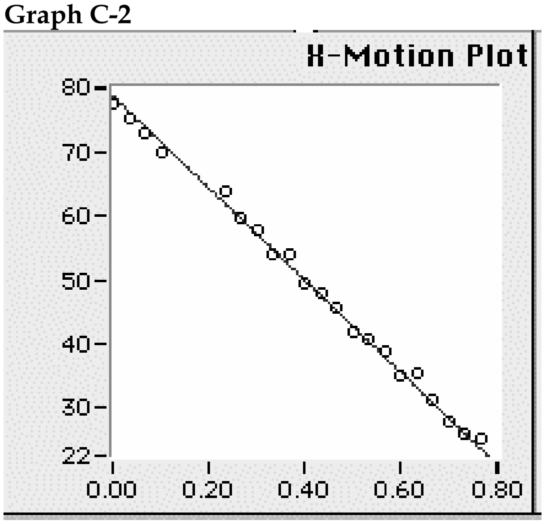
For example, if a rolling cart undergoes constant acceleration, the position-versus-time graph is curved. In fact, our theory tells us that the curve should be a parabola. To be concrete we will assume that your data starts at a time when the initial velocity of the cart was zero. The theory predicts that the motion is described by $x = (½)at^2$. To linearize this data, you square the time and plot position versus time *squared*. This graph should be a straight line with a slope of a/2. Notice that you can only linearize data if you know, or can guess, the relationship between the measured quantities involved. Professional physicists will sometimes try multiple linearizations if they see results in their data for which theoretical predictions explaining the phenomenon are unavailable. While this may seem more like prospecting than science, these guesses at linearization, if successful and repeatable (ie taking the data again), often inform and lead to new theories.

How do I interpret graphs from LabVIEW™?

Graphs C-2 through C-4 were produced with LabVIEW™. They give the horizontal position as a function of time. Even though LabVIEW™ draws the graphs for you, it usually does not label the axes and never puts in the uncertainty. You must add these to the graphs yourself.

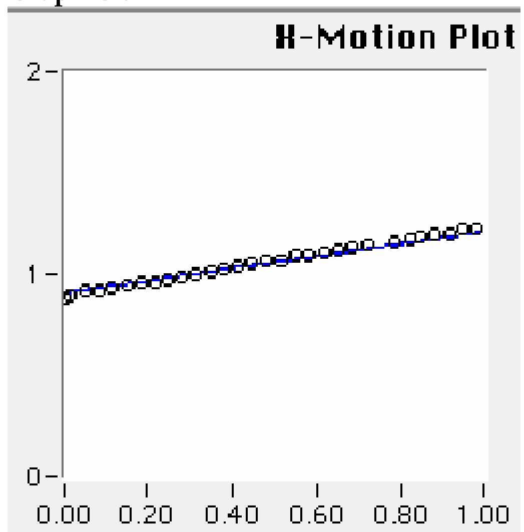
One way to estimate the uncertainty is to observe how much the data points are scattered from a smooth behavior. By estimating the average scatter, you have a fair estimate of the data's uncertainty. Graph C-2 shows position versus time for a moving cart. The vertical axis is position in cm and the horizontal axis is time in seconds. See if you can label the axes appropriately and estimate the uncertainty for the data.

Graph C-2

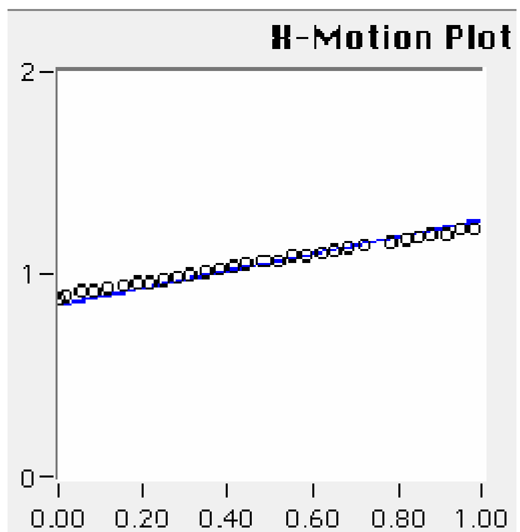


Finding the slope of any given curve with LabVIEW™ should be easy after you have chosen the mathematical equation of the best-fit curve. Graphs C-3 and C-4 each show one possible line to describe the same data. From looking at these graphs, the estimate of the data's uncertainty is less than the diameter of the circles representing that data.

Graph C-3



Graph C-4



The line touching the data points in Graph C-3 has a slope of 0.38 m/sec. This is the **minimum** possible slope for a line that fits the given data. The line touching the data points in Graph C-4 shows a slope of 0.44 m/sec. This is the **maximum** possible slope for a line that fits the given data.
You can estimate the uncertainty by calculating the average difference of these two slopes from that of the best-fit line. Therefore the uncertainty determined by graphs C-3 and C-4 is $\pm$ 0.03 m/sec.

PRACTICE EXERCISES:

Explain what is wrong with each of the graphs below: